Empirical Credit Risk

Advantages of using a market measure of credit risk.

Oren Cheyette and Boris Postler

As bond investors have entered riskier markets in recent years, there has been increasing demand for quantitative models of credit risk. We describe a simple, empirical approach to modeling bond credit risk in a portfolio context, accounting for correlations driven by common factors and idiosyncratic residuals. This data-driven model sidesteps many of the complications of more fundamental models, but nevertheless effectively incorporates the main factors driving corporate bond returns.

One popular type of credit risk model predicts default probabilities using fundamental firm data and equity prices.[1] The prototype for such structural models is given by Merton [1974]. His model treats a defaultable corporate bond as a combination of a default-free zero-coupon bond and a short put option on the firm, with strike equal to the bond's face value.

Structural models have been shown to be better predictors of default likelihood than agency ratings. Yet they are notably deficient at matching market valuations (yield spreads) of corporate bonds (see Eom, Helwege, and Huang [2002] and Huang and Huang [2003]). Moreover, structural models are dependent on structural (accounting) data, which are not always available in a timely fashion.

An alternative approach that has gained some popularity is the class of reduced-form models.[2] These models treat default as a random event, with the default probability—or, more generally, credit quality migration—either estimated from historical data or derived from observed bond spreads (or both).

A primary use of default models is for risk management by bond investors. (One use not discussed here is relative value measurement, i.e., finding alpha.) For

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mark-to-market investors for whom short- and medium-term changes in portfolio market value may be as important as ultimate repayment over the life of a bond, the change in market pricing of a bond driven by changing default expectations may be of greater practical importance for risk management than actual default. Neither of the two standard approaches to credit risk modeling reliably achieves this link between market returns and the underlying default model.

Cheyette and Tomaich [2003] describe a third approach to this problem, designed to model market risk directly, bypassing the use of probabilistic default models. Their *empirical credit risk* (ECR) method entails modeling the linkage between bond and equity market returns, providing estimates for bond-equity hedge ratios, and bringing to bear the power of equity market risk forecasts for bond risk forecasting. We provide a somewhat condensed version of these earlier results, as well as some new results and applications of the model.

A key element of the ECR analysis is to take the market price of a bond (in the form of its option-adjusted spread) *as an input*, rather than attempt to derive it from fundamental data, as structural models do. The ECR model uses the spread as a market measure of the issuer's creditworthiness. In this respect, the ECR model is similar to reduced-form models—market prices are an input to the model rather than an output. Unlike reduced-form models of credit risk, however, which model default probabilities, this is a model of return relationships.

Our research reveals some interesting facts about the links between bond and equity markets. Most important, a bond's spread relative to the Treasury curve is a good measure of the covariance between the bond's return and Treasury bond returns and with the return of the issuer's equity. While some academic studies have found a weak relationship based on agency rating, we find a strong dependence on market spread.[3] From the standpoint of modeling risk, a significant advantage of using bond spread is that, compared to agency rating, it is a much more timely and responsive measure of perceived credit quality.

Second, upon examination of a number of factors that might have additional influence on the return relationships, two effects stand out. Bond duration is found to increase the exposure of bonds to equities at all levels of credit quality, although the data are thin at higher duration for the weakest credits. We also find clear evidence that the bond-equity return linkage is significantly weaker for *positive equity specific return* than for other sources of equity return (common factors or negative issuer-specific events).[4] This is likely due to agency effects where management acts to increase shareholder value at the expense of bondholders, e.g., by announcement of spinoffs, stock repurchases, or other leverage- or volatility-increasing action.

Finally, even after accounting for corporate bond returns due to interest rates and equity returns, there remain significant cross-sectional residual returns. Although we cannot attach a fundamental explanation to these returns, they nevertheless have identifiable common components, consistent with the findings of Collin-Dufresne, Goldstein, and Martin [2001]. We therefore augment the ECR model by adding a spread factor model of the residual returns. The end result is a model capable of explaining anywhere from 45% to 90% of the cross-sectional variance (R^2) of bond returns, depending on credit quality (as measured by agency rating). This is a significant improvement on a model using only interest rate and spread factors, for which the model R^2 falls as low as 15% for the riskiest high-yield bonds.

MODEL STRUCTURE

We seek to model the return to a corporate bond in terms of the returns to government bonds—i.e., changes in default-free interest rates—and the return to the bond issuer's equity. We represent this relationship through the return decomposition:

$$r^t_{Bond} = \beta_{IR} r^t_{GovBond} + \beta_E r^t_{Equity} + \varepsilon^t_{Bond} \tag{1}$$

giving the excess return r^t_{Bond} to a corporate bond in terms of the corresponding excess return $r^t_{GovBond}$ to an equivalent government bond, the excess return of the bond issuer's equity r^t_{Equity}, and a residual ε^t_{Bond}.[5]

The equivalent government bond return is derived by treating the bond as default-free, computing the implied interest rate factor exposures $-D_{B,i}$ (yield curve durations), with the conventional minus sign, and then summing the return contributions from interest rate factor changes obtained (by inversion of this relation) from actual government bond excess returns: $r^t_{GovBond} = \sum_i (-D_{B,i}) r^t_{IR,i}$. The interest rate factors are given by approximate principal components of the term structure movements.[6]

The calculation of durations adjusts for the timing of cash flows and other bond structural features (but not default), so that when the coefficients of Equation (1) are estimated using the government bonds originally used to find the interest rate changes (and taking $\beta_E,\ r^t_{Equity} = 0$, of course), we obtain $\beta_{IR} = 1$.

The coefficients β_{IR} and β_E measure the dependence of the bond return on interest rate changes and equity returns. Ultimately, we are seeking functional representations for β_{IR} and β_E to capture their dependence on credit quality and perhaps other characteristics of the bond or firm in question.

Before looking at the data, we can anticipate some qualitative aspects of the relation between bond prices and returns, interest rate changes, and equity returns. We expect the spreads of bonds with low perceived default risk to be narrow, and, conversely, we expect market spread of bonds with high default risk to be wide.[7] Accordingly, we anticipate that returns to low-spread bonds will be primarily determined by changes in default-free interest rates, and nearly independent of equity returns, while returns to high-spread bonds will be less sensitive to changes in default-free interest rates, and more sensitive to equity returns.

In the Merton framework, a bond's value is determined by interest rates, firm value, leverage, and the volatility of firm value. Exhibit 1 shows the Merton model coefficients β_{IR} and β_E as a function of bond spread for a range of firm value volatilities.

The spread dependence of β_E has a simple explanation—higher spreads correspond to higher firm leverage, making the firm's put option more valuable to the shareholders and correspondingly more costly (in expectation) to the bondholders. The spread dependence of β_{IR} is primarily due to the fact that a higher interest rate implies a higher risk-neutral expected growth rate for firm value, reducing the value of the put option. This reduced value of the short put partly offsets the decline in bond value due to the interest rate change. The size of this effect grows with increasing leverage as the put becomes more valuable.

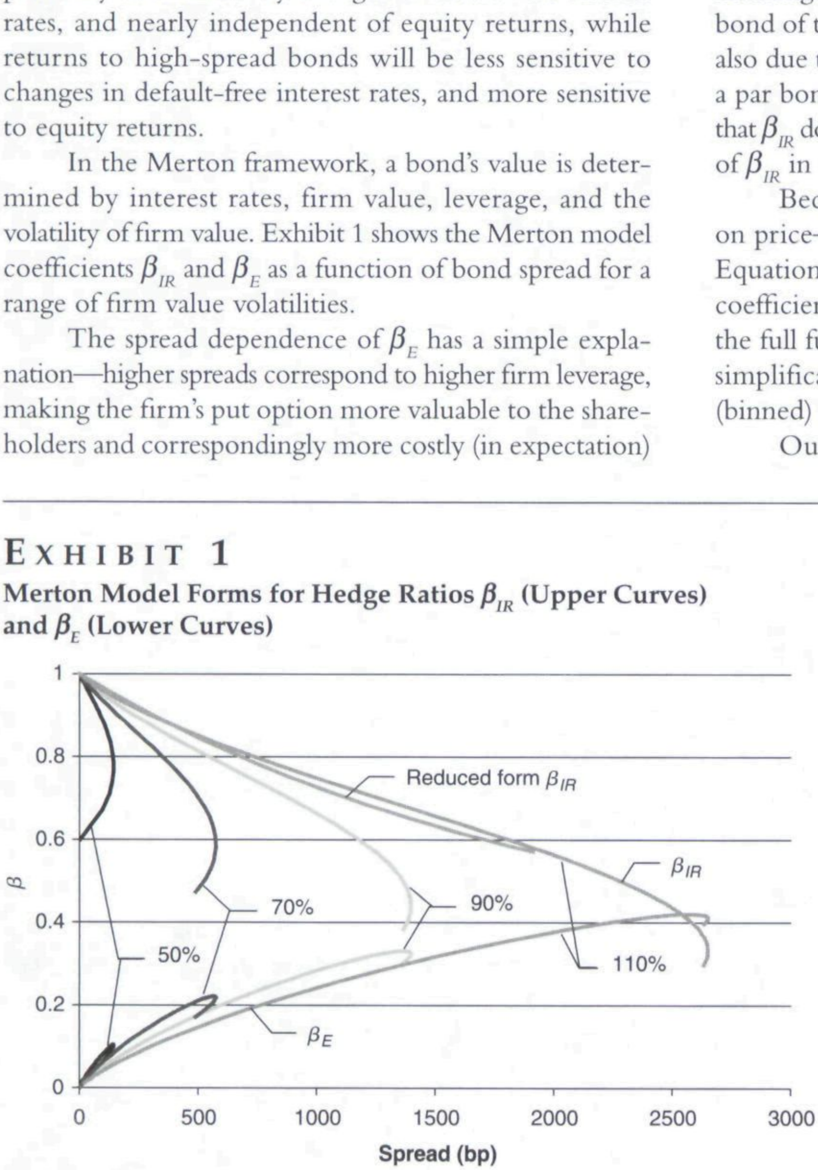
In the reduced-form framework, the default probability is not explicitly linked to leverage, so there is no implied dependence between bond and equity returns. There is, however, a spread dependence of β_{IR}. For a par bond, this is because riskier bonds have higher coupons, resulting in a lower duration than for a default-free par bond of the same maturity. With non-zero recovery, it is also due to the early receipt of part of the principal. (For a par bond, it turns out that there are cancellations such that β_{IR} does not depend on the recovery rate.) The behavior of β_{IR} in this setting is also shown in Exhibit 1.

Because β_E depends on spread—or, equivalently, on price—the bond-equity return relationship given by Equation (1) is non-linear. Equation (1) with constant coefficients may be viewed as a linear approximation of the full functional relationship. We will make use of this simplification in exploratory analyses of data grouped (binned) by spread.

Our empirical findings, although similar to the Merton model predictions, are quantitatively quite different. (β_{IR} also has an observed dependence on spread quite different from the form implied by reduced-form models.) Ultimately, for risk modeling purposes, we choose heuristic functional forms for β_{IR} and β_E based on criteria of parsimony (few parameters), smoothness, and reasonable limiting behavior at narrow and wide spreads.

EXHIBIT 1

Merton Model Forms for Hedge Ratios β_{IR} (Upper Curves) and β_E (Lower Curves)



As a function of bond spread for annual equity volatilities between 50% and 110%. Calculations assume a 5 year bond with constant 5% interest rates.

DATA

The regression Equation (1) involves three sets of returns. On the left side are bond excess returns. On the right side are equity excess returns and equivalent government bond returns, or, in practice, interest rate factor returns and calculated exposures of the bonds to the interest rate factors. Our estimation universe, then, consists of a collection of bond and corresponding equity excess returns (typically more than one bond's return per equity return) pooled cross-sectionally and over time and the corresponding interest rate factor changes for each period in the sample.

For convenience, and to minimize problems of synchronization and stale price information, we use monthly returns. Bond data are obtained from several sources. For the U.S. market, we use price data from the Merrill Lynch Index Group, representing evaluations for the Merrill Corporate Master and High Yield Master (Cash Pay) indexes. Euro and U.K. market data are combined from Merrill Lynch and Citigroup corporate indexes, giving priority to Merrill prices when both are available.

By restricting our analysis to these benchmark portfolios, we include only a relatively liquid portion of the bond universe. Bond price data are notoriously unreliable, but to the extent that we find a positive effect relating bond and equity returns, it is unlikely to be an artifact of vendor mispricing. Rather, vendor pricing errors, such as stale prices, will tend to reduce the linkage effect we are studying.

Equity data are obtained from Bridge (Reuters) and IDC. We include the estimation universe from the Barra U.S. and U.K. equity risk models and the model coverage for the Euro equity risk model (USE3L, UKE7, and EUE2, respectively). Each month, the USE3L estimation universe includes roughly 2,000 issues and the UKE7 700, while model coverage for the EUE2 includes 4,900 non-U.K. stocks. We then match bonds to equities using Reuters' company identifiers.

The data start in January 1996 for the U.S. market and in January 1999 for the euro and U.K. markets, and run through August 2004. Bonds are matched to parent company issuers only—so, for example, our analysis does not include bonds issued by finance subsidiaries of industrial firms.

One of the significant challenges in this analysis is the linking of bond and equity return data, because there is no global standard for common issuer identifiers. We have used a variety of sources and methods for constructing these linkages. The techniques are most succesful in the U.S., so that in aggregate we have approximately 138,000 matched returns for this market. For the euro zone we have 15,000 matched returns and for the U.K. 5,200. The statistical uncertainty of our results in market-dependent analyses varies accordingly.

Most of the results reported below are for the U.S. data only. Market-specific and global analyses are identified explicitly.

ANALYSIS AND RESULTS

To build a risk model, we will eventually want to estimate smooth functional forms similar to the Merton model shapes, and giving reasonable asymptotic behavior for both β_{IR} and β_E at low and high spreads. But before estimating this heuristic model, we characterize the empirical behavior of the β's we need to know what factors are responsible for variation in the dependence of bond returns on equity returns and interest rate changes, and what the smooth empirical functional forms should look like, at least in the range of spreads where the results are not too noisy.

Starting with a regression of Equation (1) on the aggregate data binned by spread range, we then drill down to examine additional dimensions of possible variation. The statistical uncertainty of the regression β's is estimated by bootstrap resampling with 200 runs. Error bars in the exhibits are one standard deviation uncertainty estimates from the bootstrap runs.[8]

We take spread as a basic market measure of credit quality that we expect to affect the regression coefficients. Spread is calculated relative to a default-free zero-coupon yield curve fitted to government bond prices, although spreads relative to a swap curve give fairly similar results.

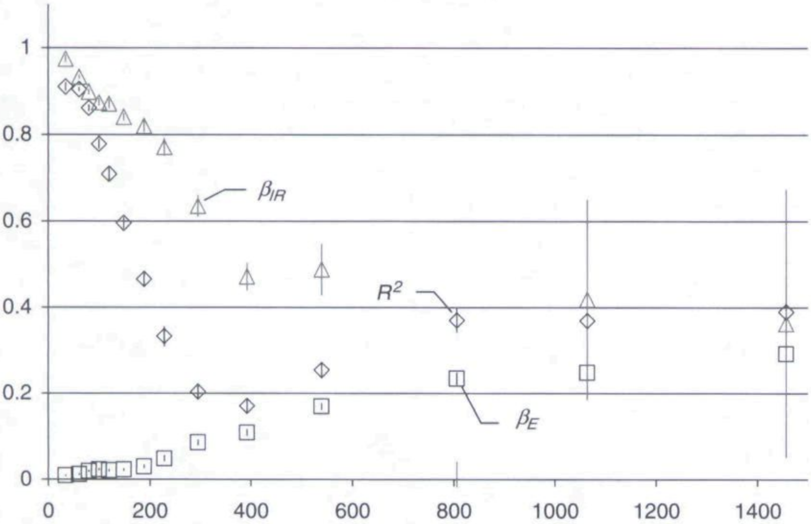
Aggregate Estimation Results

The results of the regressions on data binned by bond spread are shown in Exhibit 2. The graph shows three series of values: the interest rate β, higher on the left; the equity β, higher on the right, and the regression R^2, high on both the left and the right and falling to a minimum around a 400 basis point spread. The one standard deviation error bars are too small to see for many of the data points, primarily at low spread.

As measured by the t-statistic, β_E is significantly different from 0 at all spread levels, even in the 0-50 b.p. range. Yet equity return contributes substantially to explaining bond return only once the spread exceeds about 200 b.p. The interest rate exposure, β_{IR}, is not significantly different from 0 for the spread > 700 b.p. bins, well into high-yield territory.

One notable feature of Exhibit 2 is the minimum of the R^2 in the neighborhood of the investment-grade/high-yield boundary. For bonds with spread between about 300 and 500 basis points, the model R^2 reaches a minimum of just under 0.2. The low R^2 means that neither interest rate changes, as measured by

EXHIBIT 2
Estimation of Equation (1) on Bond Return Data



Binned by spread for U.S. market over 1/1996-8/2004.

government bond returns, nor changes in firm value, as measured by equity returns, are explaining much of these bonds' returns.

A further interesting observation is the rapid drop in β_{IR} from near 1 for the lowest spread bin to below 0.5 for bonds with spreads above 350 b.p., while for bonds with spread above 700 b.p., the estimate of β_{IR} is not significantly different from 0. The highest credit quality bonds have interest rate sensitivity nearly equal to that of Treasury bonds with the same cash flows. On the other hand, interest rates explain essentially none of the returns of low credit quality bonds. This is consistent with the market lore that duration overstates the sensitivity of lower-grade bonds to interest rates.

What is perhaps surprising is the rapid drop in β_{IR} for the higher credit quality range with spreads under 200 b.p. These are mostly investment-grade bonds, and we see already that at the lower quality end of this range β_{IR} is around 0.8, implying that the empirical duration of a nominally five-year duration bond is really closer to four years. This has obvious relevance for the problem of contructing a bond portfolio to match a duration target.

Exploratory Analysis

We expand the analysis to include additional groupings of the data by:

- Bond maturity (duration).
- Projected equity volatility.
- Sample period.
- Equity return sign and common and specific equity returns.
- Sector.
- Market.
- Market capitalization and return period.

The first two tests are motivated by considerations from structural models. In particular, we expect that bond maturity and equity volatility should influence the exposures of bonds to equity. (The main effect of maturity on interest rate exposure for high-quality bonds is already accounted for by the calculated factor exposures, $(-D_{B,i})$.)

The other tests are of sources of variation not predicted by a structural approach. Grouping by sample period tests the stability of the empirical relationship over time. Grouping into positive and negative equity return sets was originally to test whether the general phenomenon of higher correlations in down markets holds in this area. A subsequent analysis also extends Equation (1) by splitting the equity return into two components: a term due to the common-factor return, based on the USE3L equity model, and the residual, or specific equity return. This analysis tests for evidence of firm value-shifting corporate actions, such as equity buy-backs or other leverage-increasing activities.

Grouping by sector is of particular interest for financial firms. Due to their typically high leverage such companies' debt is difficult to model in a structural framework.

Grouping by market (we looked also at the U.K. and the euro zone) is of interest for obvious reasons—we want to know whether we need to estimate the model separately by market, which poses data challenges, or whether a single global model is viable.

Grouping by market capitalization of the issuer is an indirect way of looking for a liquidity or pricing transparency effect. Large-cap issuers might be expected to have more liquid bonds with well-revealed prices than small-cap issuers.

Finally, we also examine the dependence of the results on the return period (with periods up to 12 months).

Briefly summarizing the results:

1. None of the examined factors strongly affects the estimates of the interest rate exposure coefficient β_{IR}. A bond's interest rate exposure depends on bond characteristics as measured by its calculated exposures ($-D_{B,i}$) and its spread, and not on any other factor examined.
2. For spreads in the intermediate range of 100-1,000 b.p., β_E increases moderately with bond maturity or duration. This is shown in Exhibit 3. One can always express bond value changes in terms of spreads. If the spread change corresponding to a firm value change were independent of maturity, we would find β_E to be proportional to duration. We actually find weaker dependence than this, implying that—all else equal—spreads of shorter-term bonds change more than those of longer-term bonds for a given equity return. In extreme cases, this can produce spread curve inversion, as is sometimes observed for distressed issuers.
3. β_E is not significantly sensitive to sample period, equity volatility, or sector.
4. β_E depends strongly on the sign of the equity return; negative-return events have a much greater effect on bond return than positive-return events. By splitting the equity term in Equation (1) into two components, we find that this effect is confined to firm-specific returns, and is not visible in marketwide (common-factor) returns. That is, the effect appears to arise largely or entirely from firm-specific events that cause positive equity and negative bond returns. Presumably, these are value-redistributing corporate actions such as announcements of stock buy-backs, debt issuance, or takeovers.
5. The forms of β_{IR} and β_E are quantitatively similar in two other major bond markets that we have examined, the U.K. and the euro zone. Exhibit 4 shows the binned estimates of β_E for the separate markets and the parameterized estimate for the aggregate data. The results are not substantively different from results for the U.S. data. (Due to the smaller data samples, as noted earlier, the binned β estimates are much noisier in the U.K. and euro data, but qualitatively similar to the U.S. data.) We also examine data for the Japanese market, but there is very little high-yield bond data available, and we do not find any evidence that equity returns are explanatory for Japanese bond returns.
6. Both β_E and the cross-sectional R^2 of the model increase with issuer market capitalization (Exhibit 5) and the length of the return period (Exhibit 6), suggesting that even at horizons as long as a month, there is a significant amount of firm-level information remaining unrevealed in the reported bond prices for smaller issuers. (Alternative explanations of a large-cap long-horizon bias in β_E and R^2 have not occurred to us.)

One consideration in use of an empirical model is that, in the absence of a strong theoretical framework, it is hard to be confident that the model applies to a new market environment. In conversations with practitioners, this has emerged as a particular concern for application of the empirical credit risk model as bond spreads have moved from extremely high levels in late 2002 to extremely low levels in early 2005.[9]

Cheyette and Tomaich [2003] failed to find sample-period dependence of the β estimates in U.S. data for three periods ending in early 2003. Although this analysis included both high- and low-spread periods (in particular, data prior to the Russian default of mid-1998), it is interesting to see a comparison including the most recent low-spread period.

Exhibit 7 shows a comparison of β_E from regressions over a high-spread period

EXHIBIT 3

Binned Regressions and Best Fits of Smooth Functions to Data Grouped by Bond Duration (U.S. Data Only)

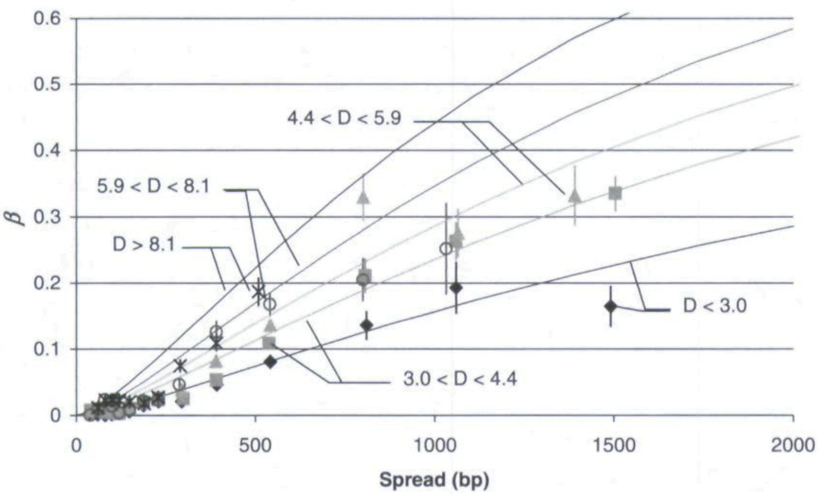


EXHIBIT 4

Best-Fit and Binned Regressions for Corporate Bonds in Three Major Markets

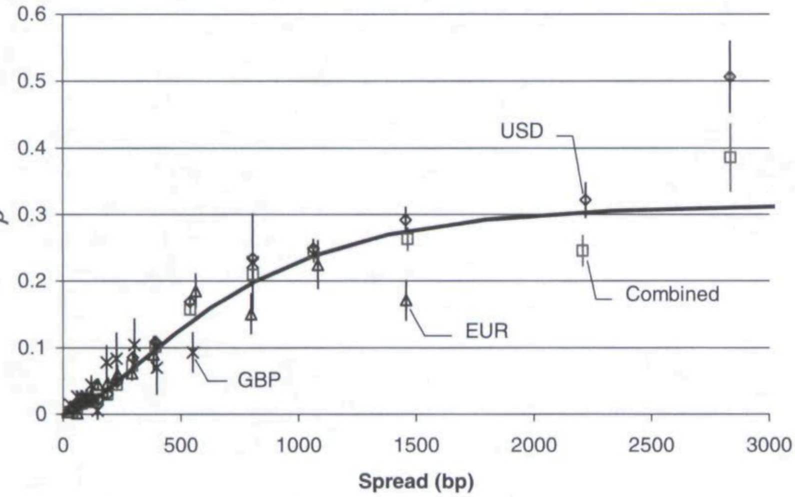
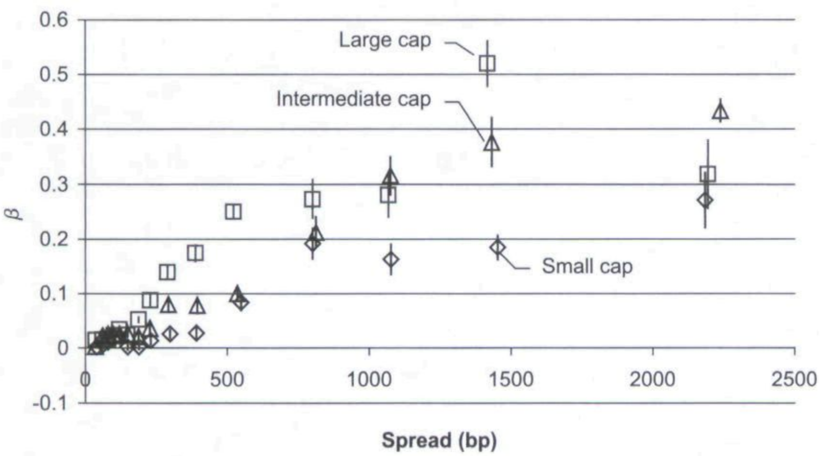


EXHIBIT 5

Binned Regression Estimates of β_E for Issuers Grouped by Market Capitalization



Issuers grouped into five cohorts for each spread level; cohorts shown are the smallest (average market cap = $3.6 billion), middle (average market cap = $16.6 billion), and largest (average market cap = $77 billion) quintiles.

early in the decade and the subsequent very low-spread period. For the high-spread period, we take an interval when our estimated average spreads for BBB bonds were over 200 b.p. This period runs from the stock market downturn in April 2000 through the subsequent bankruptcy wave, and ends in early 2003 as spreads began to compress. The low-spread period, when average BBB spreads were below 130 b.p., begins seven months later in September 2003, and runs for the remainder of the sample, through September 2005.

Exhibit 8 shows the distribution of bond spreads in the two sample periods. The distribution in the high-spread period has very few data in the lowest-spread bins and peaks at 150 b.p., while in the low-spread period the peak is at the lowest spread of 35 b.p. Although the typical spread levels in the two sample periods were quite different (as were the corporate default rates), the behavior of β_E as a function of spread level is quite similar. These results strongly suggest that the absolute level of a bond's spread is a good measure of the degree of linkage between bond and equity returns, independently of the general level of spreads.

APPLICATION TO RISK MODELING

We estimate heuristic functional forms for the regression coefficients relating bond returns, in order to apply the observed relations to various portfolio construction and measurement tasks, such as computing hedge ratios, risk forecasting, and scenario analysis.

Residual Spread Factors

Traditional models of market risk for corporate bonds are based on the attribution of bond returns to interest rate and residual credit spread factors. Interest rate factor returns, derived from either returns to government bonds or changes in swap rates, are exogenous to the corporate bond universe. Spread factors are typically derived endogenously, by categorizing credit-risky bonds as, e.g., AA-rated industrials, A-rated financials, and so on, attributing the aggregate excess return of each group to a common spread return (after removing the return due to interest rate changes).

The return decomposition for a corporate bond is given by the equation:

$$r^t_{Bond} = r^t_{GovBond} + (-D^{spd}_B)\Delta s^t_{Factor} + \varepsilon^t_{Bond} \tag{2}$$

EXHIBIT 6

R^2 of Binned Regressions for Longer-Term Bond and Equity Returns

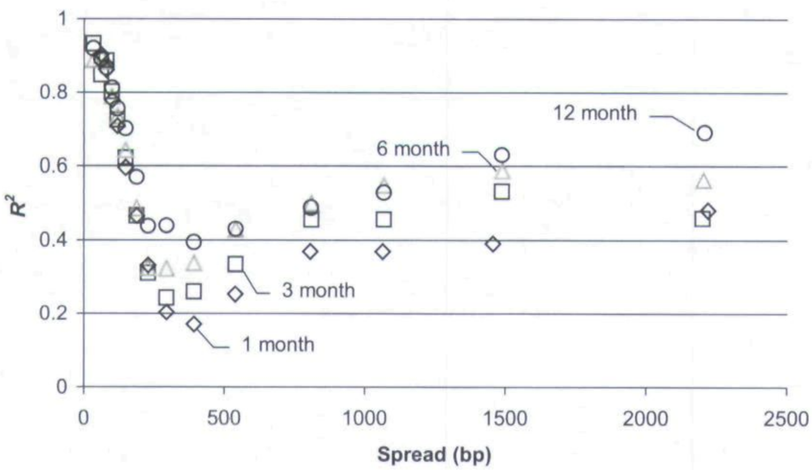
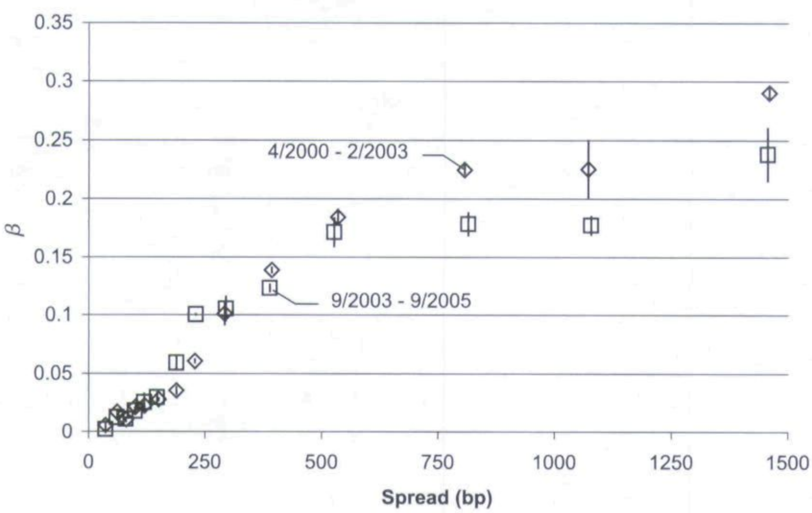


EXHIBIT 7

Estimated β_E for Bond Return Data Grouped as in Exhibit 8



where $r^t_{GovBond}$ is the equivalent government bond return, as in Equation (1); $(-D^{spd}_B)$, the negative of the bond's spread duration, is its exposure to the common-factor spread; and Δs^t_{Factor} is a market common-factor spread appropriate to the bond, e.g., BBB financial.

The spread factors s^t_{Factor} are often referred to as credit risk factors, although they are not directly due to credit risk in the sense of issuer-level risk due to downgrade or default. Rather, they account jointly for marketwide changes in default risk, the repricing of credit risk for groups of similar issuers, and other valuation effects such as discounting for liquidity risk. Historical time series of these factor returns are then used to construct forecasts of the future distribution of bond returns. We refer to the factor model of Equation (2) as the *rates plus spreads*, or RS model.

This attribution works quite well for high-grade corporate bonds, for which interest rate changes are the dominant source of cross-sectional return variation, while common spread factors account for a quarter to half of the remaining return variance. As we move down the credit quality spectrum, interest rates and common spreads account for a rapidly declining share of return.

Changes in government bond yields and common spread factors tell us very little about returns to bonds rated BB and below. After incorporating spread factors, the R^2 of a model based on Equation (2) ranges from below 40% for bonds rated BB and B to below 20% for bonds rated CCC as can be seen in Exhibit 10. The R^2 estimates for the ECR model shown in Exhibit 2, on the other hand, suggest that we can do better at explaining the sources of lower-grade bond returns, and therefore do a more effective job in forecasting the distribution of bond returns.

In order to apply the ECR return analysis to risk forecasting, we first develop an estimate for a smooth approximation of the empirical behavior of the β functions seen in Exhibit 2. For any bond, these functions will tell us how to compute its exposures to the risk factors of our interest rate and equity models, as well as its exposure to the idiosyncratic issuer risk captured by the equity-specific risk model.

The behavior shown in Exhibit 2 is reasonably well captured by the functions:

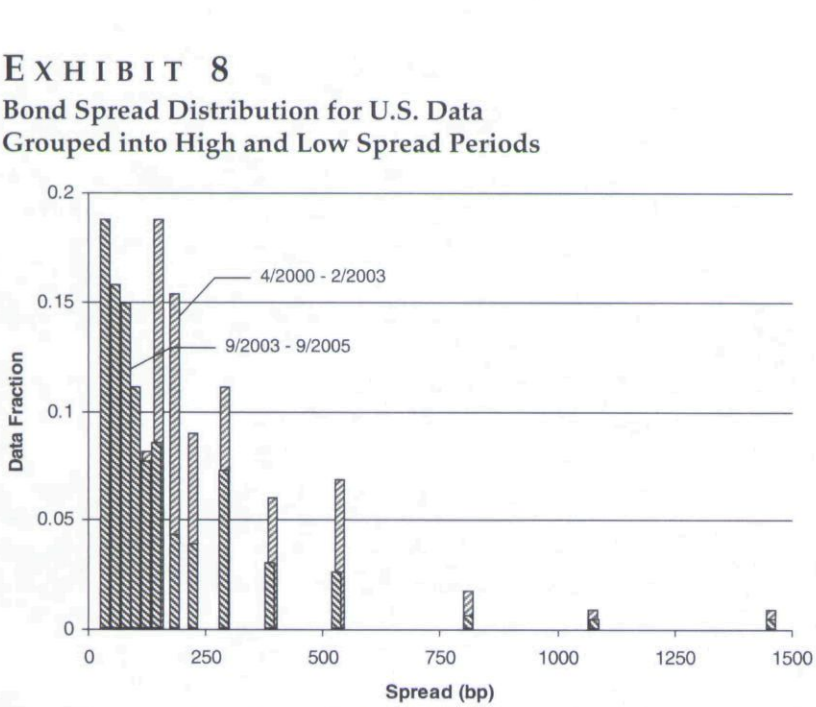
$$\beta_{IR} = 1-(1-e^{-\alpha_1 \cdot S})^{\alpha_2} \tag{3-A}$$

$$\beta_E = \alpha_3(1-e^{-\alpha_4 \cdot S})^{\alpha_5} \tag{3-B}$$

where $\alpha_1,\ldots,\alpha_5$ are parameters defining the shapes, and S is bond spread. The choice of these functional forms is somewhat arbitrary, dictated primarily by the desire that

EXHIBIT 8

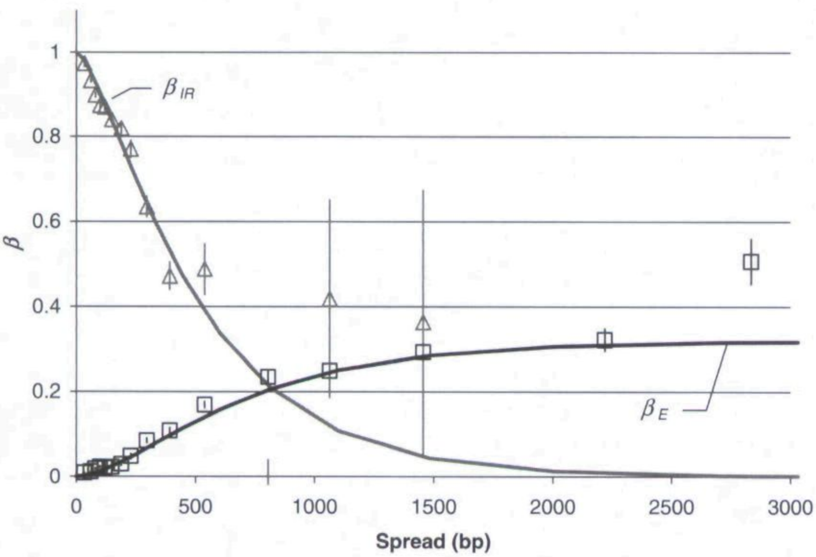
Bond Spread Distribution for U.S. Data Grouped into High and Low Spread Periods



The average bond spread is 244 bp in the earlier period, 145 bp in the later.

EXHIBIT 9

GLS Fit of Parameterized Functions (3-A, 3-B) to Aggregate Data, Compared to Binned Regressions



the functions be asymptotically constant at a wide spread, lie between 0 and 1, and be described by a small number of parameters, to avoid overfitting. (We take β_{IR} to approach zero at wide spreads, as the binned regression results are not significantly non-zero.)

We require fitted functional forms such as these, rather than simply using the results of the binned regressions for two reasons. One is that we expect the true relationship between bond and equity returns to vary smoothly and monotonically with credit quality, while binned regressions would give us discontinuous and possibly non-monotonic behavior (due to statistical errors). Second, at high spread levels we have relatively few data for estimation of β_{IR} and β_E, so the estimates will be quite noisy. By imposing functional forms with reasonable limiting behaviors, we can obtain usable estimates for the values at this extreme, as well as smooth interpolation at lower spread levels.

We are also free to adapt the heuristic functional forms of β_{IR} and β_E to accommodate dependence on additional characteristics of bonds or firms. For example, we found that β_E depends on duration as well as spread, and therefore in application we modify Equation (3-B) to

$$\beta_E = \alpha_3\left[(1-e^{-\alpha_4 \cdot S})(1-e^{-\alpha_6 D \cdot S})\right]^{\alpha_5} \qquad \text{(3-B}'\text{)}$$

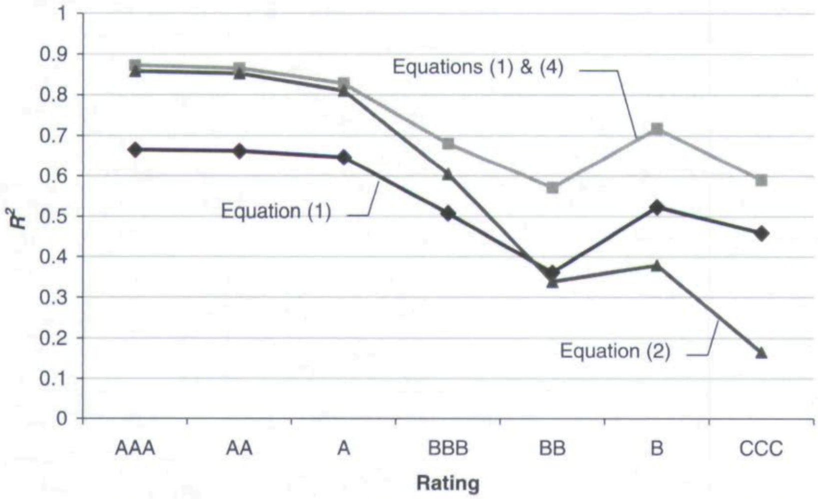
where D is the bond's spread duration.

Exhibit 9 shows the functions β_{IR} and β_E of Equations (3-A) and (3-B) with parameters estimated by general least squares from the same data as the binned regressions of Exhibit 2 (included for comparison). The simple functional forms clearly provide reasonable smooth interpolation and extrapolation of the empirical bond return relationships.

Exhibit 10 shows the explanatory power of the ECR model with the estimated functional forms of Equations (3-A) and (3-B′), and of Equation (2), the RS model, grouping the data by rating. Although the ECR model does a better job of explaining returns to bonds rated BB and lower, the conventional RS model of Equation (2) outperforms the ECR model on this measure for investment-grade bonds rated BBB and better.

Given that the sector/rating spread factors contribute roughly 0.1 to 0.2 of improvement in R^2 for intermediate- to lower-grade bonds in the RS model, we extend the

EXHIBIT 10
Fraction of Bond Return Variance (R^2) for Three Bond Return Attribution Models



Bonds grouped by rating category. Results are for U.S. data from January 2000 through October 2002.

ECR model by looking for additional factors to explain the residuals from Equation (1). These residuals are regressed on a spread model where each bond is exposed to a spread factor based on sector and rating, with magnitude equal to minus the spread duration.

This is very similar to the method used to estimate the RS model, except that rather than fitting the spread model to the residuals after accounting for interest rate changes alone, the spread model is now applied to the residuals after accounting both for interest rate changes (with β_{IR}-scaled exposures) and equity returns. The resulting model of returns can be written as a two-stage regression.

Given the residuals ε^t_{Bond} from Equation (1), we estimate a second regression:

$$\varepsilon^t_{Bond} = (-D^{spd}_B)\Delta s^t_{Factor} + \eta^t_{Bond} \tag{4}$$

where Δs^t_{Factor} is the spread change for the issuer's sector/rating combination, and η^t_{Bond} is the remaining residual return. Note that the spread changes Δs^t_{Factor} are not changes to market spreads for bonds in each sector, first, because a portion of each bond's spread change has already been accounted for by the equity return, and second, because the interest rate exposures are scaled by $\beta_{IR} < 1$. We refer to these as spread changes because the bonds' exposures to them are equal to their spread durations, and because for high-grade bonds, for which $\beta_{IR} \sim 1$ and $\beta_E << 1$, they may be close to the conventional sector/rating spread change.

The added explanatory R^2 produced by the second regression is also shown in Exhibit 10. The calculations are based on the regression formulas using the heuristic functional forms of Equations (3-A) and (3-B′). By adding a spread model for the bond residual returns of Equation (1), we obtain a return decomposition that performs well across the credit spectrum.

Risk Analysis and Stress-Testing

Given the β functions (with the fitted parameters), along with standard factor covariance forecasts for interest rates, equities, and the residual spreads, we are equipped to provide risk forecasting and scenario analysis for corporate bonds.

According to Equations (1) and (4), a corporate bond will be exposed to both bond market and equity market factors:

- Interest rate factors, such as key rates or principal components, with exposures equal to the conventionally computed exposures (e.g., key rate durations) multiplied by β_{IR}.
- One or more spread factors, depending on the factor breakdown chosen for the residual credit spread changes, with exposure $(-D^{spd}_B)$.
- The bond issuer's equity risk factors, with exposure scaled by β_E.

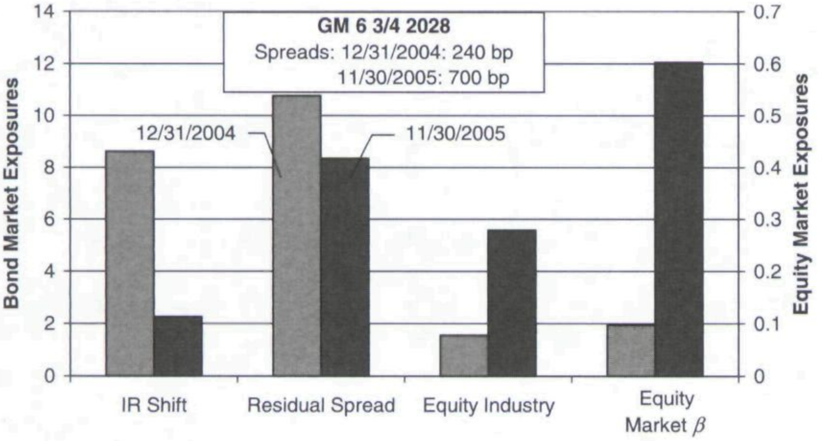
In addition, a bond will have issuer-specific risk from its exposure to equity-specific risk and also a residual contribution from the volatility of η^t_{Bond} in Equation (4).

Exhibit 11 shows the resulting exposures for a General Motors bond with remaining time to maturity of about 25 years at the end of December 2004, when its spread was a relatively low 240 b.p., and the end of November 2005, when its spread had widened to 700 b.p. By the parameter estimates for Equations (3-A and 3-B′), we obtain at the earlier date 0.8 for β_{IR} and 0.078 for β_E. The interest rate exposures are comparatively high and the equity exposures correspondingly low. At the later date, with wider bond spread, we obtain instead 0.27 for β_{IR} and 0.28 for β_E.

The consequence of this for risk forecasts is shown in Exhibit 12. The upper and lower charts show the total risk and risk decomposition forecasts for this bond on the earlier and later dates of Exhibit 11, according to both

EXHIBIT 11

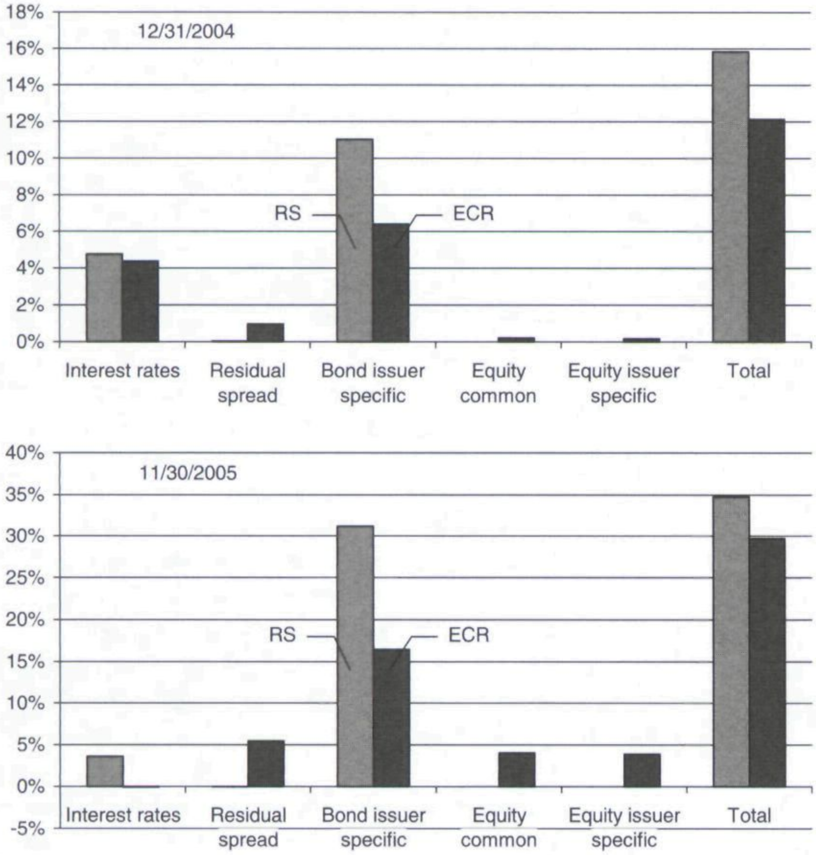
Market Risk Factor Exposures of GM Bond at Beginning and End of 2005



Yield curve parallel shift and residual spread exposures are on the left, equity industry exposure and predicted equity market β are on the right.

EXHIBIT 12

Risk Forecast (Annualized) and Attribution for a Long Term GM Bond at Beginning and End of 2005



the RS and the ECR model. (The risk decomposition uses the Litterman method of attributing half of each covariance term to each of the contributing factors.) Risk forecasts are obtained using the Barra Integrated Model, combining bond and equity factors and specific risk estimates.

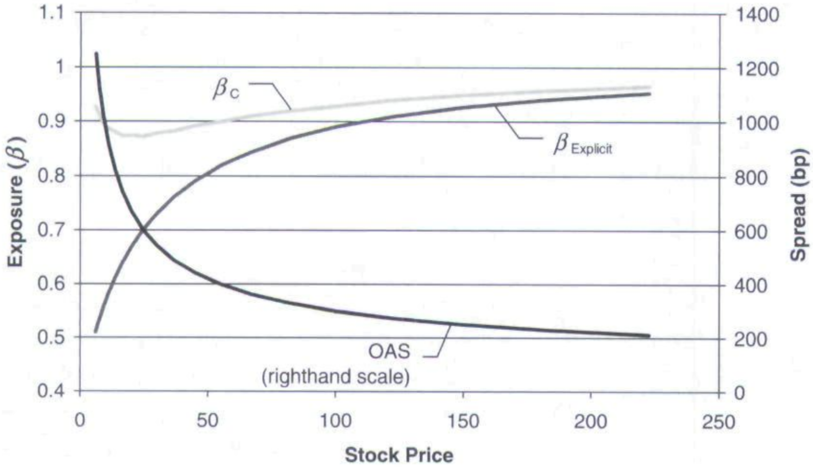
The total risk forecasts of the two models are not substantially different, but the explanation in terms of risk factors and issuer-specific volatility is very different at the later date. In the non-ECR framework, the risk is due almostly entirely to bond issuer-specific spread volatility; because of the high observed negative correlation of spreads and interest rates, the interest rate risk and spread common-factor risk net to a fairly small total contribution, attributed here to interest rates. In the ECR framework, bond issuer-specific volatility continues to provide a significant contribution to total risk (just over half), but the combined contribution of the equity factors is also substantial, and the overall contribution from common factors is appreciably greater.

An implication of the ECR model is that a bond's risk can be partially hedged with a short position in the issuer's stock, with hedge ratio β_E. In the case of the GM bond of Exhibits 11 and 12, β_E ranges between 0.2 and 0.3 for most of 2005. Measured relative to the bond market value, the realized return volatility for a portfolio long \$1 of the GM bond and short \$ β_E of the GM stock (adjusted daily) from July 1, 2005, through January 3, 2006, was 23.4% versus 25.5% (annualized) for an unhedged bond position—a difference of 8%. At the start of that period, the ECR model predicts annualized volatility of 21% for the hedged portfolio and 23.3% for the unhedged bond, a difference of 10%. In this case at least, the ECR prediction is borne out quite accurately.

The ECR model also makes interesting predictions for convertible bonds (CBs). In the ECR framework, a CB has two sources of exposure to the issuer's equity: the explicit exposure due to the conversion option, and

EXHIBIT 13

ECR Model Prediction for Equity Exposure of Convertible Bond



According to Equation (5).

the implicit exposure arising from credit risk. A simple argument gives the combined equity exposure β_C of a CB as:[10]

$$\beta_C = \beta_{Explicit} + \frac{D_S^C}{D_S^B}\beta_E(s, D_S^B) \qquad (5)$$

where $\beta_{Explicit}$ is the (calculated) explicit exposure, D_S^C and D_S^B are the spread durations of the CB and a non-convertible bond with otherwise equivalent terms, and $\beta_E(s, D_S^B)$ is the ECR model equity exposure of the bond.

If a firm's credit quality deteriorates after it issues a convertible bond, $\beta_{Explicit}$ will (most probably) fall as the stock price drops (although an increase in the equity implied volatility could offset the drop to some degree). At the same time, β_E will increase as the bond spread widens; the ratio D_S^C / D_S^B will generally also increase, as the conversion option is less likely to be exercised. The net result is that the bond's exposure to the equity will fall less than predicted by models that account only for the explicit option component.

The predicted effect can be considerable, as seen in Exhibit 13. While the calculated equity exposure of the convertible drops rapidly as the stock price drops below the conversion price, the increasing contribution from the credit exposure keeps the combined exposure near 1.0.

Exhibit 13 illustrates another application of Equation (1), scenario analysis. Scenario-based risk modeling involves creating a set of simulated asset returns based on a factor model or a historical distribution of returns, or some other method (e.g., stress-testing). In any case, we can use Equation (1) to estimate scenario returns for corporate bonds, given the equity returns.

If we include only the equity return effect (that is, leaving aside interest rate and residual spread changes), we can integrate Equation (1) to obtain:[11]

$$\int_{B_i}^{B_f} \frac{dB}{B\beta_E(s(B))} \approx \log(1 + r_E) \qquad (6)$$

where B_i and B_f are the starting and ending bond prices, $s(B)$ is the bond spread as a function of bond price, and r_E is the equity return.

With the further approximation that the spread duration, $D_B^S = -\frac{\partial \log B}{\partial s}$, is independent of bond price, we obtain the simplification:

$$\int_{s_i}^{s_f} \frac{ds}{\beta_E(s)} \approx -\frac{1}{D_B^S}\log(1 + r_E) \qquad (7)$$

Equation (7) is used to derive the spread curve of Exhibit 13, solving iteratively for the ending option-adjusted spread s_f of the bond, given its initial spread s_i, as a function of the stock return. Of course in practice the other sources of spread variation also contribute (and there is the path-dependence to worry about as well), so Equation (7) must be interpreted as a useful approximation, appropriate when the equity return is high compared to other factors.

SUMMARY

We have developed an empirical model of the return linkage between a corporate issuer's bonds and equity, based on analysis of U.S., euro, and U.K. bond and equity returns. A key ingredient in the model is use of a bond's market spread over the government bond yield curve to parameterize the regression coefficients (β_E and β_{IR}) relating the bond's return to the issuer's equity return and to interest rate changes.

Bonds' exposures to the issuer's equity become significant when their spreads exceed 200 to 300 basis points, a level characteristic of the boundary between agency ratings considered investment-grade and speculative (high-yield). Bonds' exposures to interest rates decline rapidly in the same spread range.

In an analysis pooled over time and spread level, we find that very high-quality corporate bond returns are explained almost entirely by interest rate changes, which account for close to 90% of return variance. Equity returns account for 40% or more of the return variance of low-grade bonds, those with spreads above around 800 b.p.

Interestingly, returns to bonds of intermediate credit quality do not appear to be significantly explained by either exogenous source. Although we cannot explain the underlying drivers of return for these bonds, we nevertheless can identify common factors similar to sector/rating spreads, which raise the overall R^2 of the model to above 50% for all levels of credit quality. We also observe that the estimates of interest rate and equity exposure for a given spread level are stable over time, comparing a period of unusually high spreads (2000–early 2003) to a later period of exceptionally low spreads (late 2003–late 2005).

We examine several dimensions of variation in bond and equity attributes to find additional factors affecting β_{IR} and β_E. We do not find an economically significant effect for most of the factors, but three do have an impact.

The equity exposure of higher-duration bonds increases more rapidly with spread than that of lower-duration bonds.

There is a substantial difference in β_E between positive and negative equity excess return events. On further analysis, this difference appears to be confined to positive equity-*specific* returns, which are much more weakly linked to bond returns than are negative specific returns or common-factor returns of either sign. A possible explanation attributes the phenomenon to managerial actions that shift firm value from bondholders to shareholders. Such actions will always be intended to produce positive equity-specific return. Whether this is the full explanation for the observations remains an open question.

There is a significant positive dependence of β_E and model R^2 on both issuer capitalization and the length of the return period. We interpret both effects as attributable to price transparency. Larger-cap issuers are better monitored (because they have more frequent bond issues and more issues outstanding), so the prices of their bonds are updated more rapidly by our data vendors. Return period-dependence is due, we believe, to the slower adjustment for smaller issuers of reported bond prices to market events.

The empirical credit risk model provides an alternative means to analyze and forecast corporate bond credit risk. By using a market measure of credit risk we are able to sidestep many of the complications associated with more fundamental models, while still capturing the main determinants of corporate bond returns.

ENDNOTES

[1]Commercially available models include Barra's Default Probabilities and Moody KMV's EDFs.

[2]See Jarrow, Lando, and Turnbull [1997] and Duffie and Singleton [1999] for references.

[3]See, e.g., Kwan [1996], Alexander, Edwards, and Ferri [2000], Collin-Dufresne, Goldstein, and Martin [2001], Hotchkiss and Ronen [2002], and Treptow [2002].

[4]Equity specific return, also called idiosyncratic return, is the component of a stock's return not explained by either passage of time (the risk-free rate) or market common factors.

[5]*Excess return* is here defined as total return minus the risk-free rate, not total return minus benchmark return, as is sometimes done for equity factor models.

[6]Barra's interest rate risk models incorporate three factors (approximate principal components of the covariance matrix of spot rates) responsible for most of the term structure variation in the 3-month to 30-year maturity range. An alternative specification would be in terms of spot rate changes at standard maturities. The results would be unaffected, as the three principal components account for more than 98% of the variance of the spot rates.

[7]The spread is the amount by which the default-free yield curve must be shifted in order that the bond valuation model reproduces the bond's market price. Our valuation algorithm captures contributions to bond value due to cash flow timing and embedded options, such as calls, puts, and sinking funds, but assumes no default. This definition generalizes the notion of a yield spread of option-free bonds relative to a default-free benchmark bond.

[8]See Efron and Tibshirani [1993].

[9]Barra's estimate for U.S. intermediate BBB spreads peaked at 280 b.p. in October 2002, and reached a low of 80 b.p. in February 2005.

[10]The dependence of the CB return on the stock return can be written as $\frac{d \log C}{d \log S} = \frac{\partial \log C}{\partial \log S} + \frac{\partial \log C}{\partial \log B}\frac{\partial \log B}{\partial \log S}$, where C is the CB price, S is the stock price, and B is the non-convertible bond price. At a fixed interest rate, $\partial \log C / \partial \log B = D_S^C / D_S^B$, while from Equation (1) $\partial \log B / \partial \log S = \beta_E$.

[11]Generically, Equation (1) describes a path-dependent relationship between equity and interest rate changes and the bond return, because the β functions are not components of an exact differential. This complication goes away if we restrict attention to bond returns implied by equity returns alone.

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